Chapter 8

Interest rate swaps

An interest rate swap (IRS) is an OTC interest rate derivative contract in which two parties enter into a reciprocal obligation to exchange interest coupons in the same currency during an agreed term without exchanging principals. Interest rate swaps are often used to change the interest rate conditions of a financial instrument, usually from fixed to floating or vice versa.

IRS maturity periods vary from one to fifty years. The principals differ from 100,000 to 100 million or even more. The reference rate for the floating interest coupon of an IRS in euro is usually the three or six-month EURIBOR rate. For IRS contracts in other currencies, this is usually a LIBOR rate.

The fixed IRS rate is determined by supply and demand on the IRS market and usually follows the market interest rate for government bonds with a spread. The fixed-interest is usually set for the entire term of the IRS. In some cases, both interest rates are floating. This is the case, for instance, for an IRS in which a three-month EURIBOR is exchanged for a one-year EURIBOR. An IRS involving the exchange of two floating interest rates is called a basis swap.

8.1 Contract specifications and jargon

The following transaction data is recorded in an IRS contract:

- the nominal value or principal
- the reference rate for the floating interest rate and the daycount convention (e.g. EURIBOR or LIBOR, actual/360)

- the contractual interest rate; the level of the fixed interest rate and the daycount convention
- the term and any possible repayment schedule
- who the buyer is and who the seller is; the fixed rate payer and fixed rate receiver

If parties regularly enter into financial contracts, a framework agreement is often concluded. For IRS contracts, this is a BBAIRS general master agreement, drawn up by the British Bankers' Association or an ISDA general master agreement. An ISDA agreement regulates the following items:

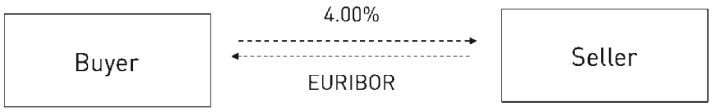
- definitions regarding the instruments and the settlement (terms and conditions and payment procedures)
- applicable law
- procedures for cancellation
- the extent to which bilateral contracts can be transferred to other parties (assignment)
- who is authorised to enter into transactions
- what information the parties must provide for the other parties and how frequently this must happen
- how transactions are confirmed.

For the party that pays the fixed rate in an IRS, the swap is a payer's swap. For the party that receives the fixed rate, the same IRS is a receiver's swap. Sometimes the terms buying or selling an IRS are used. As usual, the general rule with regard to buying and selling in financial markets applies: a buyer profits from an increase in the price determining parameter and a seller profits from a decrease in the price determining parameter, in this case the fixed interest rate. The buyer of an IRS is thus the one who pays the fixed rate because he profits from a rise in interest rates.

The term for IRS contracts can vary from only several days to fifty years. The principal sum can vary widely. For most transactions, it varies between EUR 1 million and EUR 100 million. The reference for the floating interest rate in a euro IRS is generally the 3 or 6 month EURIBOR rate. These swaps are then also called EURIBOR swaps. For IRS contracts in other currencies, this is usually a LIBOR rate.

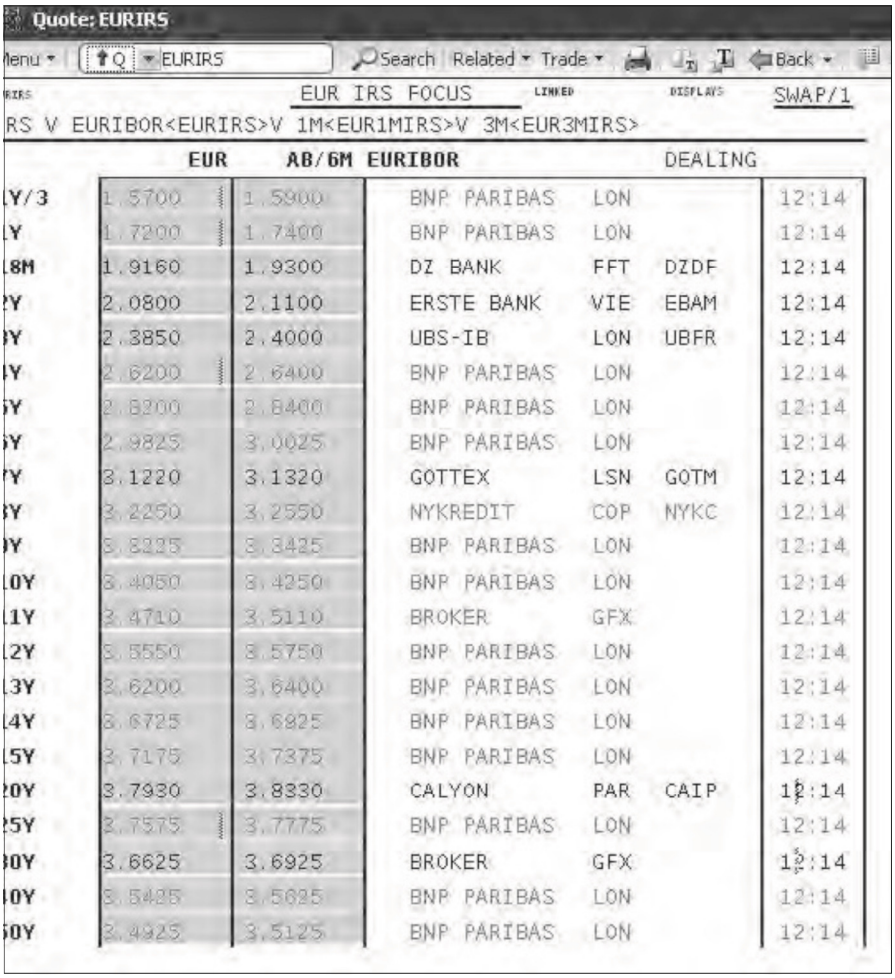
Figure 8.1 shows a diagram of an IRS. The buyer pays the fixed interest coupon and the seller pays the floating interest coupon.

Figure 8.1 ***Interest rate swap***



The 'price' or IRS rate for a swap is the fixed rate level which is determined by supply and demand and is often indicated as a spread with respect to the market rate for government bonds. The fixed rate is usually set for the entire term of the IRS. Figure 8.2 shows a Thomson Reuters screen with IRS prices.

Figure 8.2 ***IRS prices***



8.2 Settlement of an IRS

Both the fixed and floating coupon in an IRS are normally paid in arrears. The frequency for the floating coupon is usually quarterly or semi-annually. Except for Great Britain, two days prior to the expiry of each floating coupon period, the new floating coupon rate is set for the next coupon period. The fixed coupon is generally paid at the end of each year. Often, on the payment date of the fixed coupon, the amount of the fixed coupon is netted against the floating coupon that is due on the same date (payment netting). This is a standard arrangement for an ISDA contract.

All interest rate fixings and coupon payments of an IRS are recorded in an event calendar. The table below shows the event calendar for a receiver's IRS with a contract term from 15/7/2009 until 17/7/2012 for which the reference interest rate is the six-month EURIBOR. The IRS is concluded under an ISDA framework agreement, therefore, the fixed coupon is netted against the last floating coupon of the year.

DATE	EVENT
13/7/2009	Fixing 6 month EURIBOR
	Coupon period 15/7/2009 - 15/1/2010
13/1/2010	Fixing 6 month EURIBOR
	Coupon period 15/1/2010 - 15/7/2010
15/1/2010	Paying of floating coupon
	Coupon period 15/7/2009 - 15/1/2001
13/7/2010	Fixing 6 month EURIBOR
	Coupon period 15/7/2010 - 15/1/2011
15/7/2010	Receiving of net amount of fixed coupon of 1st year and floating coupon for coupon period 15/1/2010 - 15/7/2010
13/1/2011	Fixing 6 month EURIBOR
	Coupon period 15/1/2011 - 15/7/2011
15/1/2011	Paying floating coupon
	Coupon period 15/7/2010 - 15/1/2011
15/7/2011	Receiving of net amount of fixed coupon of 2nd year and floating coupon for coupon period 17/1/2011 - 15/7/2011

In ISDA framework agreements it is sometimes agreed that coupon payments should be fully synchronised. This means that the coupon frequency for the fixed leg is set to be the same as that for the floating leg and that the fixed rate is converted to the daycount convention of the floating rate (actual/ 360). The fixed rate must then be adjusted in two ways:

- from annual coupon to semi-annual or quarterly coupon
- from 30/360 to actual/360.

8.3 Overnight index swaps

An overnight index swap (OIS) is an OTC interest derivative in which, for a short period, two parties undertake to exchange interest payments in the same currency without the exchange of principal sums while the floating coupon is based on an overnight interest rate index. The following transaction data must be recorded in an OIS contract:

- principal sum
- term
- level of the fixed interest rate
- reference rate for the overnight interest rate
- fixed interest daycount convention
- who the buyer is and who the seller is; the fixed rate payer and fixed rate receiver.

The maximum term for an OIS is two years. The principal sums are always large, at least EUR 10 million. Overnight indices for euro are EONIA, determined by the European Banking Federation, and EURONIA, determined by the WMBA (Wholesale Markets Brokers' Association). The difference between the two is that EURONIA is concerned with transactions concluded via brokers in London and that EONIA is concerned with the overnight transactions of the EURIBOR panel banks.

The overnight index for Pound Sterling is SONIA (Sterling overnight index average) that is determined by the WMBA, for CHF it is the CHF tom / next indexed swap rate (TOIS) and for the US the reference rate is the Fed funds rate.

Both the fixed coupon and the floating coupon for an OIS are paid in arrears using payment netting. Thus, there is only one cash flow that is due at the end of the term. The diagram below shows when this net amount is settled in the various money markets.

CURRENCY	SETTLEMENT DATE
GBP	maturity date (M)
JPY, EUR, CHF	M + 1
US	M + 2

The floating coupon is calculated using compound interest by using the following equation.

$$\text{Interest amount} = \text{nominal} \times ((1 + 1/\text{yb} \times r_1) \times (1 + 1/\text{yb} \times r_2) \times \ldots - 1)$$

EXAMPLE

On Monday, a trader buys an OIS swap in euro with a principal of EUR 100 million. The fixed rate is 0.95%. The EONIA fixings are

DAY	FIXING
Monday	0.95%
Tuesday	0.98%
Wednesday	1.01%
Thursday	1.02%
Friday	1.04%

The floating coupon can be calculated as follows[51]:

EUR 100 mio x ((1+1/360 x 0.95) x (1+1/360 x 0.98) x (1+1/360 x 1.01) x (1+1/360 x 1.02) x (1+3/360 x 1.04)-1) = EUR 19,668.07.

The amount of the fixed coupon is EUR 100 mio x 7/30 x 0.0095 = EUR 18,472.22.
The net amount to be received by the trader at the maturity date on Tuesday (M+1) is: EUR 19,688.07 - EUR 18,472.22 = EUR 1,195.85.

In the above example, the Friday fixing rate is multiplied by a factor 3/360 in stead of 1/360. This is because the Friday fixing rate applies for the whole weekend.

51 Use the Y%CUM equation in your HP financial calculator to calculate the compounded interest rate: D1= 1, Y%1 = 0.0095, B = 360, D2 = 1, Y%2 = 0.0098, D3 = 1, Y%3 = 0.0101, D4 = 1, Y%4 = 0.0103, D5 = 3, Y%5 = 0.0104. Solve for Y%CUM.

8.4 Trading and arbitraging using interest rate swaps

Interest derivative traders trade, among other things, in IRS contracts. They speculate on future developments of the long term interest rate. Traders can use IRS contracts in various ways for trading and arbitraging purposes.

8.4.1 Trading interest rate swaps

If an interest derivative trader expects an interest rate rise, he buys an IRS. If he expects a fall in interest rates, he sells an IRS.

If an interest derivative trader wants to close a single IRS position, he will conclude an offsetting IRS with the same principal and remaining term. Since an IRS is an over-the-counter traded instrument, this does not have to take place with the same counterparty. Both IRS contracts will continue to exist until the maturity date, which, for instance means, that the coupon payments of both contracts will actually continue to take place during the remaining term of the contracts. The (unrealized) trading result, however, was fixed at the moment that the trader has closed his position and will immediately be shown in his profit and loss report.

EXAMPLE

An interest derivative trader expects a rise in interest rates. He, therefore buys a ten year IRS with a fixed interest rate coupon of 3.0%. After three months the ten year interest rate has indeed risen. Based on the current market conditions, the trader has now changed his opinion and thinks that the long term interest rate will not rise any further. He will now close his position by selling an IRS for the remaining term: 9 years and 9 months. The fixed rate for this IRS is 3.35%.

The trader now has locked in a profit of 0.35% for the remaining term of the IRS contracts. This unrealized profit will be shown immediately in the trader's profit and loss account but will only be materialized during the remaining term of the IRS contracts: each year the trader will receive a fixed coupon of 3.35% over the nominal amount while he only has to pay a fixed annual coupon of 3.00%.

During the three months that the trader held an open position, he also realized an interest result. This is because he had to pay the fixed rate of 3% and was receiving the three month reference rate.

ANTICIPATING CHANGES IN THE SHAPE OF THE IRS CURVE

An IRS can be regarded as a strip of FRAs, each with the same contract rate, the IRS rate. If a trader expects that the shape of the IRS curve is going to change, he can anticipate this by taking a position in specific forward periods. If he thinks that the long term interest rates will rise while the short term rates will stay the same or may even fall (and thus that only the forward yields that lie in the far future will rise), he must buy only FRAs with a long term. If he choses to buy an IRS with a long term, however, he would buy all the composing FRAs in the IRS and not only the ones that lie the most in the future. Therefore, he also simultaneously has to sell an IRS with a shorter term. By doing this, he has a closed position in the short term FRAs and a long position in the FRAs with the longer terms. If after some time he proves to be right and the long term forward yields have indeed risen, he may close his position by buying the short term IRS and selling the long term IRS.

EXAMPLE

The current two year IRS rate is 3% and the current three year IRS rate is 3.6%. A trader expects that the forward interest rates between two and three years will fall. Therefore, he buys an IRS with a contract period of two years and sells an IRS with a contract period of three years.

After three months, he proves to be right and the relevant forward rates have indeed fallen. The one year and nine months interest rate has risen to 3.30% and the two year and nine months rate has risen to 3.65%.

The remaining term of the payer's swap now is one year and nine months and the remaining term of the receiver's swap now is two years and nine months. The trader can close his position by selling an IRS with a term of one year and nine months and by simultaneously buying an IRS with a term of two years and nine months. He has now made a profit that is composed of the following elements.

- Net accrued interest revenue for the difference in the fixed rate of the receiver's swap and the fixed rate of the payer's swap: the difference between 3.60% - 3.00% over the period for which trader has held his position (three months);
- Increase in market value of the purchased IRS with the original term of two years from zero to positive. i.e. the discounted difference between 3.30% - 3.00% over the remaining term of one year and nine months;
- Decrease in market value of the sold IRS with the original term of three years from zero to negative, i.e. the discounted difference between 3.65% - 3.60% over the remaining term of two years and nine months.

TAKING PROFIT OF ANOMALIES IN THE YIELD CURVE

Sometimes there appears a so called 'spike' in the IRS curve. This means that the interest rate for a certain period deviates from what might be expected when looking at the rest of the curve. If the spike is downward, an interest derivative trader can take the opportunity to construct a strategy whereby he concludes the following IRS contracts:

- He sells an IRS with a term that is equal to the adjacent shorter period that is shorter than the period for which the 'spike' appears
- He buys an IRS with the term for which the spike appears for twice the nominal amount
- He sells an IRS with a term that is equal to the adjacent longer period.

Once the IRS curve has taken its normal shape, the trader can close this position by concluding the opposite interest rate swaps. His total result will be made up of the change in the market value of the interest rate swaps that the concluded and an accrued interest component.

EXAMPLE

The current two year USD IRS rate and the current three year USD IRS rate are 3.00% and 3.60% respectively. The current two and a half year IRS rate, however, is 3.40%. This means there is an upward spike in the curve for a maturity of two and a half years.

A trader wants to take profit from this anomaly and sets up the following strategy:

- He buys an IRS with a maturity of two years for a nominal amount of USD 100 million at a rate of 3.00%;
- He sells an IRS with a maturity of two and a half years for a nominal amount of USD 200 million at a rate of 3.40%;
- He buys an IRS with a maturity of three years for a nominal amount of USD 100 million at a rate of 3.60%.

After three months (91 days), the IRS has regained its normal shape. The curve, however, has also risen in a parallel way. The current rates are:

1 year and nine months:	3.40%
2 years and three months:	3.70%
2 years and nine months:	4.00%

At this moment, the trader can close his position by selling an IRS with a term of one year and nine months (nominal amount 100 million), buying an IRS with a term of two years and three months (nominal amount USD 200 million) and selling an IRS with a term of two years and nine months. He has now made a profit that is composed of the following elements.

- Net accrued interest revenue over the period for which he held his position (91 days):
  interest paid: USD 100 mio x 91/360 x 3.00%
  interest paid: USD 100 mio x 91/360 x 3.60%
  interest received: USD 200 mio x 91/360 x 3.40%

- Gain as a result of the increase in market value of the purchased IRS with the original term of two years from zero to positive, i.e. the difference between 3.40% - 3.00% (= 40 basis points) over the remaining term of one year and nine months over the nominal amount of USD 100 million;

- Loss as a result of the decrease in market value of the sold IRS with the original term of two and a half years from zero to negative, i.e. the discounted difference between 3.70% - 3.40% (= 30 basis points) over the remaining term of two years and three months over the nominal amount of USD 200 million;

- Gain as a result of the increase in market value of the purchased IRS with the original term of three years from zero to positive, i.e. the discounted difference between 4.00% - 3.60% (= 40 basis points) over the remaining term of two years and nine months over the nominal amount of USD 100 million.

As a result, the trader has gained a spread of 10 basis points over a nominal amount of USD 100 million during an average period of two and a half years.

8.4.2 *Arbitrage between IRS and FRAs or STIR futures*

An IRS with a short term can be constructed synthetically by a strip of FRAs or STIR futures. For instance, a synthetic payer's swap can be composed of a strip of purchased FRAs or a strip of sold STIR futures. The interest rate for a synthetic IRS can be calculated from the rates for the successive FRAs or STIR futures. For a strip of STIR futures, however, the future prices must first be converted to implied forward yields. The rate for a synthetic IRS with a term of up to one year can theoretically be calculated by using the following equation:

Compounded interest rate = ($\prod$ (1 + d_i / year basis x r_i) - 1) x year basis / days total

where:

r_1	= LIBOR/EURIBOR
r_2 .. r_n	= FRA contract rates or implied future interest rates
d_i	= number of days in the underlying periods
days total	= term of the synthetic swap in days

The price of the synthetic swap is compared to the actual IRS rate to determine if an arbitrage opportunity exists.

EXAMPLE

For December, the following rates are known

	RATE	# DAYS
3 month EURIBOR	2.23% (2.26% bond basis)	91
MAR Eurodollar future	97.65	90
JUN Eurodollar future	97.52	90
SEP Eurodollar future	97.34	90
DEC Eurodollar future	97.23	90
1 yrs IRS (bond basis)		2.49 - 2.51%

The rate of a synthetic IRS with a term of 1 year is[52]:

(1 + 90/360 x 0.0226) x (1 + 90/360 x 0.0235) x (1 + 90/360 x 0.0248) x (1 + 90/360 x 0.0266) – 1 = 0.0246.

A trader can arbitrage by constructing a synthetic payer's swap with a term of one year by selling Mar, Jun and Sep Eurodollar futures against a composed fixed rate of 2.46% and entering into a receiver's swap with a term of one year in which he receives 2.49%.

52 Use the Y%CUM equation in your HP financial calculator to calculate the compounded interest rate: D1= 90, Y%1 = 0.0226, B = 360, D2 = 90, Y%2 = 0.0235, D3 = 90, Y%3 = 0.0248, D4 = 90, Y%4 = 0.0266, D5 = 0, Y%5 = n.a., Solve for Y%CUM.

8.5 Applications of interest rate swaps for clients of the bank

Interest rate swaps have many possible applications for clients of the bank. As an example, pension funds use them to perform their asset and liability management and companies mostly use them to change the interest rate term of individual loans.

8.5.1 *Fixing the interest on loans with a floating rate*

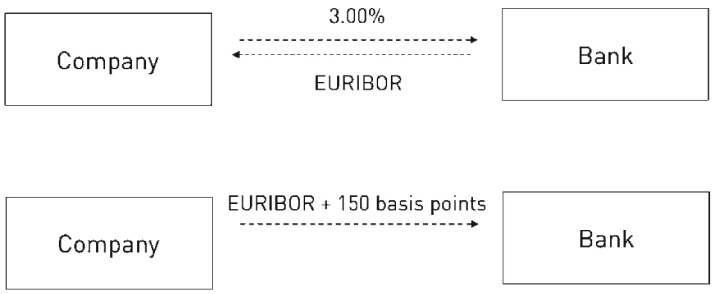
Interest rate swaps are often used by organisations that want to convert a floating interest rate in a loan into a fixed rate. These swaps are called funding swaps or liability swaps. The combination of the floating rate loan and the IRS is a synthetic fixed rate loan. With the conclusion of a funding swap, these organisations do not need to renew the existing loan or make changes to the existing loan agreement. An IRS, in itself, is a standing contract and, therefore, the IRS also does not need to be concluded at the bank that granted the loan.

EXAMPLE

A company has a loan with a floating interest rate. The interest rate is set at the 3 month EURIBOR with a credit spread of 1.50% (actual/360). The remaining term for the loan is four years.

The company wants to cover its interest rate risk for the remaining term of the loan and buys an IRS. The four year IRS rate is 3% (30/360, annual coupon).

Figure 8.3 Combination of a loan with a floating rate and an IRS



As a result, the company pays a total interest rate of 3.00% + 1.50% = 4.50%.

When the swap is concluded based on full synchronisation, the fixed rate is adjusted as follows.

CONVERSION	CALCULATION
From 30/360 to actual/360 From annual to semi-annual	3.00% x 360/365 = 2.961% $(1+0.02961)^{1/4} - 1) \times 4 = 2.93\%$
Total interest rate as a result of the combination of the loan and the IRS	2.93% + 1.50% = 4.43%

8.5.2 Fixing the floating rate of an investment / asset swap

Investors sometimes use swaps to convert a fixed interest coupon from a bond into a floating interest coupon. The combination of the fixed rate bond and the IRS is a synthetic floating rate note and is referred to as an asset swap.

Figure 8.4 *Synthetic floating rate note (asset swap)*

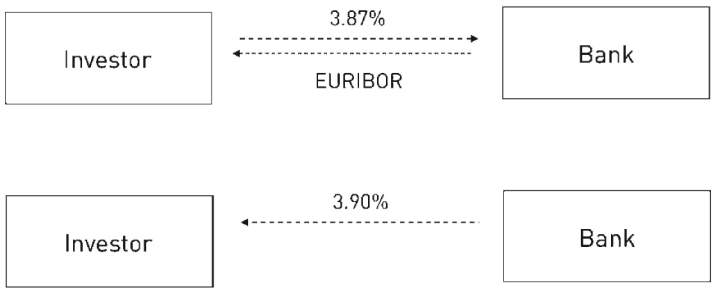
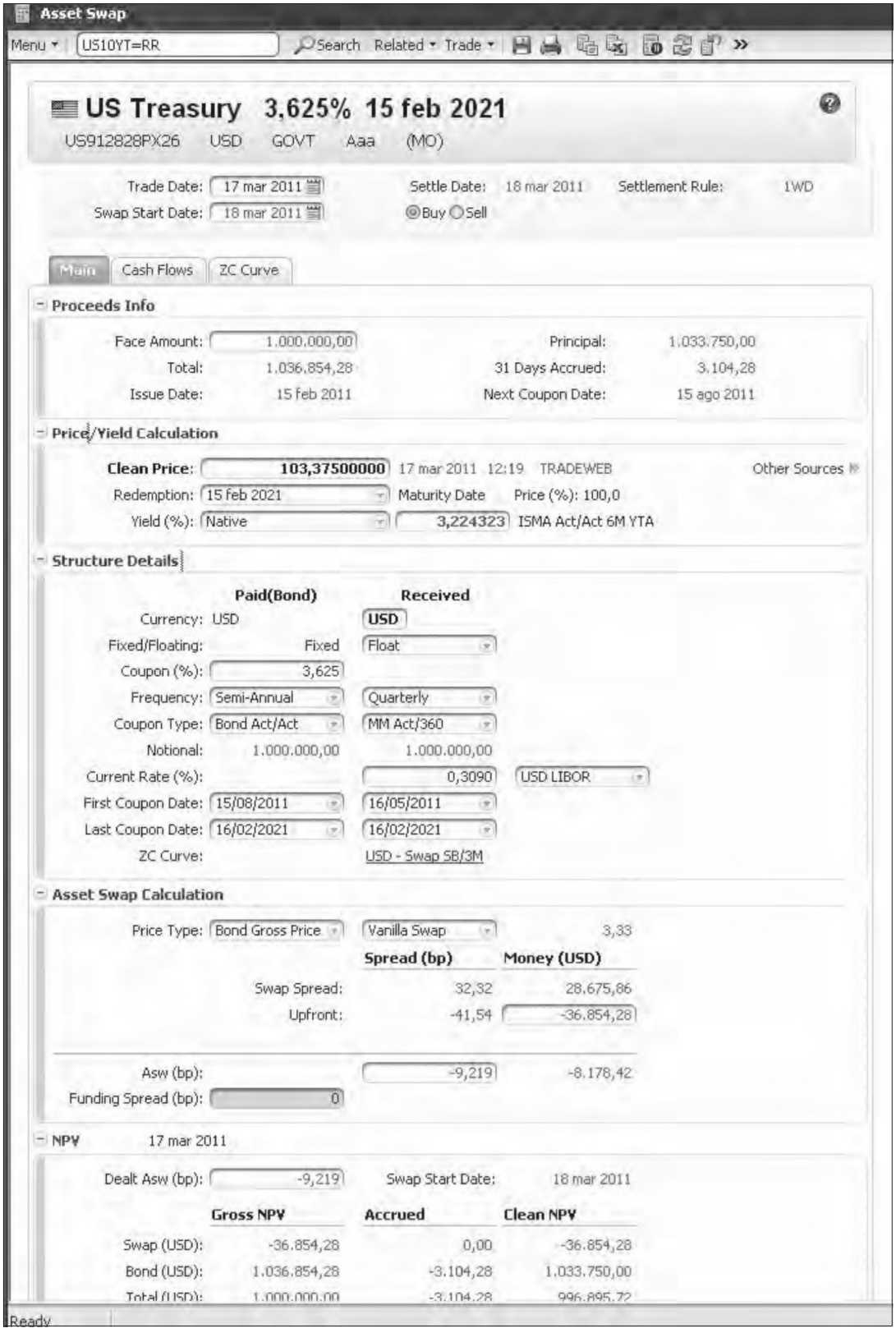


Figure 8.5 shows a Thomson Reuters pricing tool for asset swaps.

Figure 8.5 *Asset swap pricing tool*



8.5.3 *Changing interest terms*

Organizations sometimes want to change the interest rate term of a loan or an investment. For instance, in periods when interest rates are low, companies often want to extend the interest rate term of their loans. As an example, figure 8.6 shows how, in principle, a company can extend the interest rate term from three years to ten years. The company pays a fixed rate of 5.05% for the remaining term of the loan of three years. The current three-year IRS rate is, for example, 2.90% and the current ten-year IRS rate is 3.60%.

Figure 8.6 *Extending the interest rate term using par interest rate swaps*

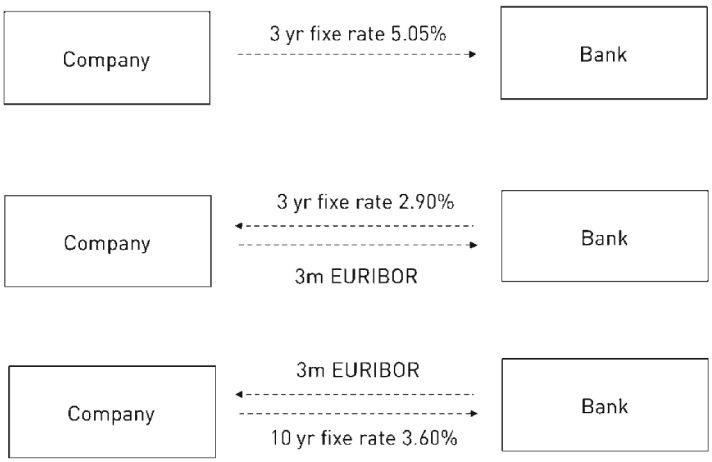


Figure 8.6 shows, however, that when the company concludes par IRS contracts (i.e. swaps with a fixed rate that is equal to the current market rate), interest costs will increase in the first three years rather than decrease. To prevent this, both interest rate swaps should be concluded at off-market prices.

The fixed rate of the three-year receiver's swap is then set to the level of the fixed rate of the loan: 5.05%. As a consequence, the company receives a spread of 2.15% over the principal sum for a period of three years. The sum of the present values of this spread is 611 basis points of the principal amount. This is shown in the following table.

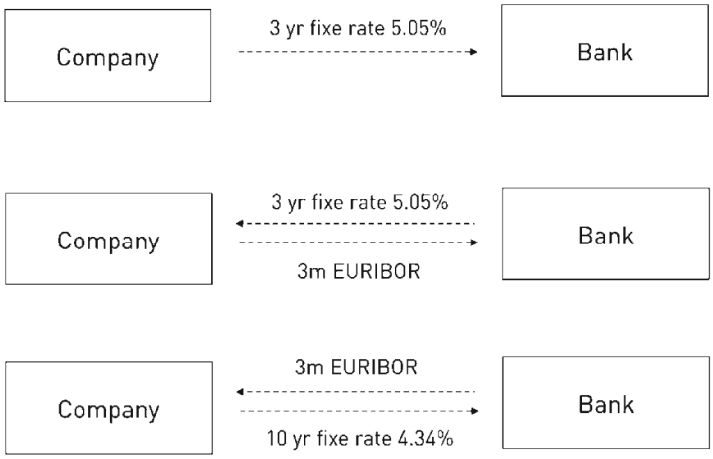
PERIOD	ZERO COUPON RATE	DISCOUNT FACTOR	PRESENT CASH FLOW (BASIS POINTS)	PRESENT VALUE CASH FLOW (BASIS POINTS)
1	0.024931	0.975675	215	209.77
2	0.026831	0.948423	215	203.91
3	0.029031	0.917729	215	197.31
4	0.031431	0.883566		
5	0.034131	0.845517		
6	0.034931	0.813826		
7	0.035631	0.782645		
8	0.036031	0.753387		
9	0.036231	0.725923		
10	0.036324	0,699917		
			Sum PV	610.99

The present value of the spread in the three years receiver's swap is then priced into the fixed rate of the ten-year payer's swap. For this, an annuity is calculated over ten years that has the same total present value as the spreads from the three-year swap (611 points). This is shown in the following table.

PERIOD	DISCOUNT FACTOR	FUTURE VALUE CASH FLOW (BASIS POINTS)	PRESENT VALUE CASH FLOW (BASIS POINTS)
1	0.975675	73.2	71.42
2	0.948423	73.2	69.43
3	0.917729	73.2	67.18
4	0.883566	73.2	64.68
5	0.845517	73.2	61.89
6	0.813826	73.2	59.57
7	0.782645	73.2	57.29
8	0.753387	73.2	55.15
9	0.725923	73.2	53.14
10	0,699917	73.2	51.23
		Sum PV	610.99

The required spread on the payer's swap is 73.2 basis points, rounded up to 74 basis points. Therefore, the fixed rate of the payer's swap is then set to 4.34% and the company now pays an effective fixed interest rate of 4.34% for a period of 10 years. This is shown in figure 8.7.

Figure 8.7 *Extending the interest rate term using off-market priced interest rate swaps*



The loan agreement and the contract for the receiver's swap both end after three years. The payer's swap, however, will still remain for a further seven years. The company must now conclude a floating rate loan in which the coupon dates correspond with those of the floating leg of the payer's swap. In this way it will continue to pay the fixed rate of 4.34% for the remaining term of the IRS.

8.5.4 *Synchronising the interest conditions of an IRS*

The conditions for an IRS must correspond with those of the instrument to which it has been concluded as a hedge, sometimes referred to as the hedged item. This applies to the term, the principal sum, the reference interest rates and - in the case of the synthetic fixed rate loan - the coupon dates of the floating leg.

In order to determine the effective interest rate for the combination of the hedged item and the IRS, any differences in the coupon frequency and/or the daycount convention should be taken into account. Many bonds, for example a U.S. T-bill, have a semi-annual coupon while the fixed interest rate for an IRS concluded under an ISDA contract has an annual coupon. Differences in the daycount convention, however, typically do not cause problems. This is because long-term rates either have daycount convention 30/360 or actual/actual. On an annual basis, this almost never leads to differences.

EXAMPLE

An investor wants to convert the fixed rate of a U.S. T-Bill into a floating rate. The semi-annual coupon of the T-bill is 3.86%. The fixed rate in the IRS is 3.87.

In order to determine the interest rate level for the synthetic FRN, the fixed rate in the IRS must be converted from an annual coupon to a semi-annual coupon[53]:

$(1 + 1/2 \times 0.0387)^2 - 1 = 0.0383$.

The interest rate level for the synthetic FRN is EURIBOR + 3 basis points.

8.5.5 *Forward starting interest rate swaps*

Sometimes, a client needs an IRS that starts in the future; this is referred to as a forward start IRS. The reason for this can be that the client already knows that he needs to take up a long term loan in the future and wants to fix his interest rate. The forward rate of a forward start swap that starts within one year can be calculated by using the following equation[54]

$$\text{forward rate} = \left(\frac{(1 + r_l)^{\text{dl/year basis}}}{(1 + d_s/\text{year basis} \times r_s)} \right)^{\text{year basis/dfw}} - 1$$

The forward rate of a forward start swap that starts after one year can be calculated by using the following equation[55]:

$$\text{forward rate} = \left(\frac{(1 + r_l)^{\text{dl/year basis}}}{(1 + r_s)^{\text{ds/year basis}}} \right)^{\text{year basis/dfw}} - 1$$

53 Use the Y%N equation in your HP financial calculator to calculate the equivalent semi-annual rate: Y% = 0.0387, N = 2, Solve for YN%

54 This equation is not part of the ACI Diploma curriculum

55 This equation is not part of the ACI Diploma curriculum

EXAMPLE

A client wants to buy a forward start IRS with a term of two years that will start after six months (183 days). The 6 month rate is 5.40% and the 2.5 year rate is 6.05%.

The fixed rate for this forward start IRS can be calculated as follows:

$$\text{forward rate} = \left(\frac{(1 + 0.0605)^{2.5}}{(1 + 0.054 \times 183/360)}\right)^{\frac{1}{2}} - 1$$

$$\text{forward rate} = \left(\frac{1.158182}{1.02745}\right)^{\frac{1}{2}} - 1 = 0.0617$$

8.6 Special types of interest rate swaps

Interest rate swaps are available in many variations. A number of these are identified below.

Accreting swap	Swap in which the principal amount increases during the term
Amortizing swap	Swap in which the principal amount decreases during the term
Roller coaster swap	Swap in which the principal amount goes up and down during the term
Swap in arrears	Swap in which the floating rate is fixed at the end of a coupon period
Callable swap	Swap that can be unwound by the buyer without any costs
Putable swap	Swap that can be unwound by the seller without any costs
Deferred start swap	Swap in which the starting date lies in the future
Extendable swap	Swap in which the term can be extended
Circus swap	Swap in which two reference interest rates in two different currencies are exchanged (e.g. US LIBOR vs EURIBOR)
Zero coupon swap	Swap in which all fixed coupons are paid simultaneously at the maturity date of the swap using compounded interest (as with the fixed coupon of an OIS swap)
Rate capped swap	Swap in which the floating rate is capped at a pre-agreed level
Constant maturity swap	Swap in which two floating coupons are paid. The reference rate of at least one of the legs is an interest benchmark rate with a term longer than one year

8.7 The valuation of interest rate swaps

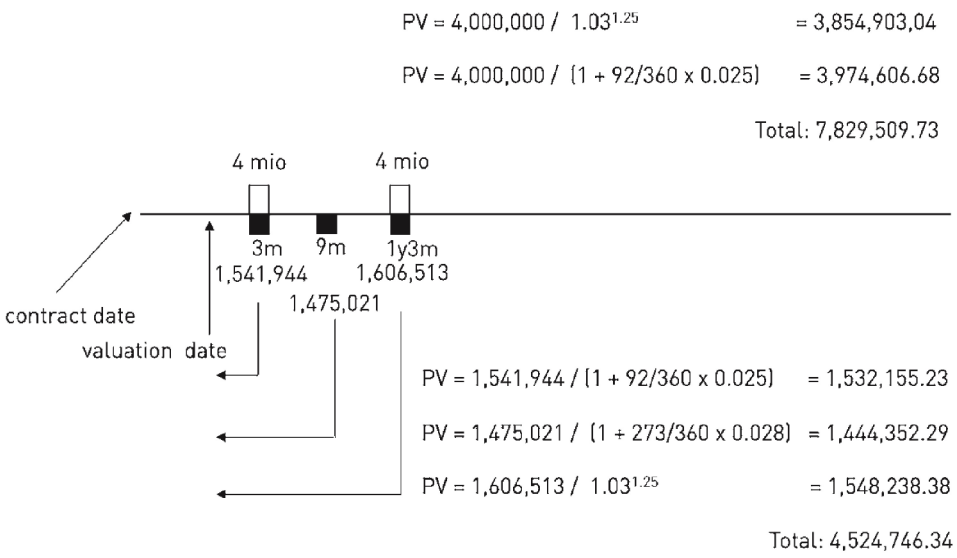
There are several ways to calculate the value of an IRS.The first way is to determine the value of an IRS is by calculating the present value of the cash flows. Figure 8.8 shows a diagram of an IRS with a principal sum of 100 million in which the fixed interest rate of 4% is received and the six-month floating interest coupon is paid. The original term of the swap was two years and the remaining term of this IRS is one year and three months. The last six-month fixing was three months before the valuation date and set to 3.05%. The size of the next floating interest coupon is thus known, EUR 100,000,000 x 0.0305 x 182/360 = EUR 1,541,944.45. The number of elapsed days for this coupon is 90.

On the valuation date, the following interest rates apply:

3-month interest rate: 2.50% (actual/360) / 92 days;
9-month interest rate: 2.80% (actual/360) / 273 days
1 year and three months zero coupon rate: 3.00% (actual/360) / 1.25 years
3s vs 9s forward rate: 2.9337% (actual/360) / 181 days
9s vs. 15s forward rate: 3.1432% (actual/360) / 184 days

Using the forward rates, the value of the future floating coupons can be calculated. Next, the present value of all future interest coupons is calculated.[56]

Figure 8.8 *Present value of the cash flows of a receiver's swap*



The dirty market value of the fixed leg is the sum of the present values of the future fixed coupons. The dirty market value of the floating leg is the sum of the present values of the future floating coupons. The dirty market value of the IRS is the difference between the (dirty) market value of the fixed leg: EUR 7,829,509.73 and the dirty market value of the floating leg: 4,524,746.34, thus EUR 3,304,763.40.

56 The amounts of the second and third floating coupon in figure 8.8 are calculated as follows:
100 mio x 181/361 x 0.024337 = 1,474,999
100 mio x 184/360 x 0.031432 = 1,606,524
The differences are caused by roundings.

The dirty market value of an IRS is used for determining the credit risk on the counterparty in the swap contract, for collateral purposes and for determining the payment in the event that the swap is unwound. Traders, however, use the clean market value of the swap to determine the value of their position. For this purpose, the dirty market value of the IRS is adjusted with the accrued interest to determine the clean market value. For the above IRS, this adjustment is as follows and the clean market value of the IRS can be calculated as EUR 1,067,245.39.

	DIRTY MARKET VALUE	ACCRUED INTEREST	CLEAN MARKET VALUE
fixed leg	7,829,509.73	EUR 100 mio x 0.04 x 270/360 = EUR 3 million	4,829,509.73
floating leg	4,524,746.34	EUR 100 mio x 0.0305x 90/360 = EUR 762,500	3,762,264.34
net value	3,304,763,40		1,067,245.39

A rough indication of the clean market value of an IRS can also be calculated by comparing the cash flows belonging to the fixed leg of the swap with that of a par IRS. With this method, that is used in the ACI exams, the amount of the current coupon period should be adjusted for the remaining term. Figure 8.9 shows how this works for the above mentioned IRS. Note that the current coupon yield is 2.9925 (this is equivalent to a zero-coupon yield of 3.00%).

Figure 8.9 *Quick and dirty method for calculating the clean market value of an IRS*

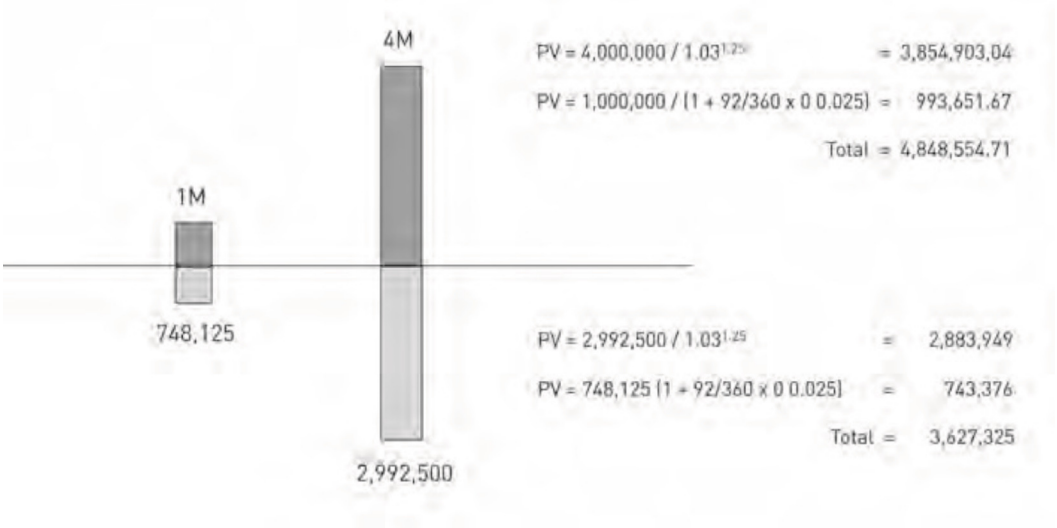


Figure 8.9 shows that the first fixed coupon is only taken into account for the remaining period of three months. According to this method, the clean market value of this swap is EUR 1,221,230. The difference with the earlier calculated clean value of EUR 1,067,245.39 is EUR 153,884.61. This difference is mainly caused by the fact that we did not include the value of the current floating coupon. This value can be calculated by the following formula:

$$\frac{100 \text{ mio} \times (0.0305 - 0.0250) \times 92/360}{1 + 92 / 360 \times 0.025} = \text{EUR } 139{,}663$$

The IRS in our example has a short remaining term. Therefore, the market value of the floating coupon is relatively high. With longer remaining terms, the value of the floating coupon plays a less important role. Furthermore, on every fixing date of the floating coupon, the value of the floating coupon will be zero.

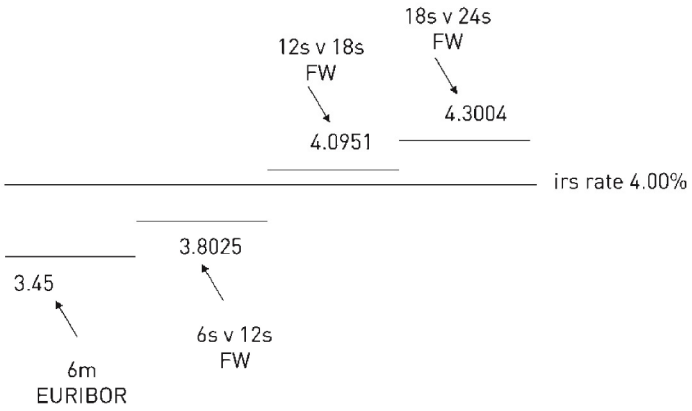
The remaining difference of EUR 14,221.61 is caused by the fact that the accrued interest is discounted, which results in the fact that the two methods only give the same outcome at a payment date of the fixed coupon[57].

MARKET VALUE OF AN INTEREST RATE SWAP AT THE START DATE

At the start date of the above swap, the two-year IRS rate was evidently 4%. Using this rate, the fixed cash flows were established: a positive cash flow of EUR 4 million after one year and a positive cash flow of EUR 4 million after two years. The value of the first floating coupon was also established. This was determined by the EURIBOR fixing at the moment of closure. The value of the second, third and fourth floating cash flow was estimated using the forward rates at that time. The IRS rate, the 6-month EURIBOR and the implied forward yields at the moment of closure are shown in figure 8.10.

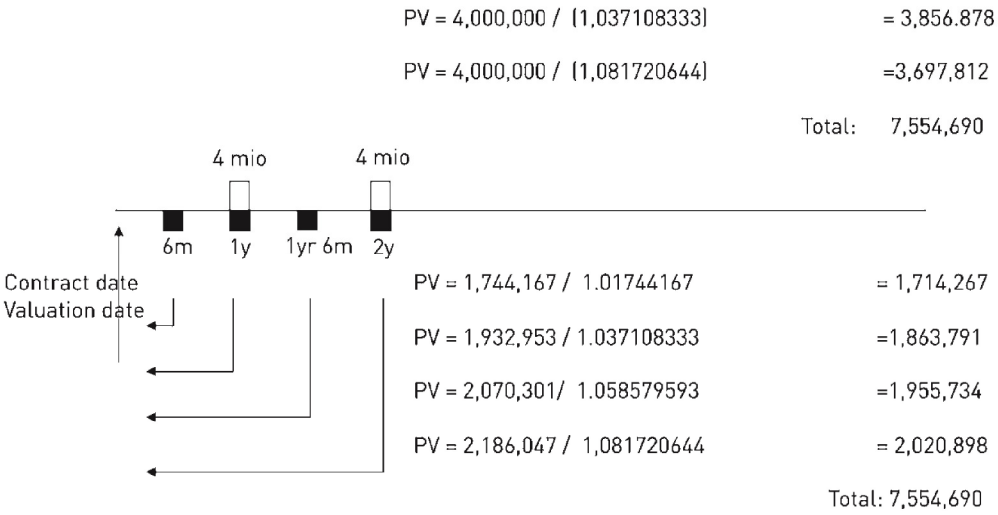
57 The accrued interest for the next fixed coupon is EUR 3,000,000. If this amount had been discounted at a discount rate of 2.5% , this would have been only EUR 2,980,955 which is EUR 19,045 lower. The accrued interest for the floating coupon is EUR 762,500. If this amount had been discounted at a discount rate of 2.5% , this would have been only EUR 757,660, which is EUR 4,840 lower. The net result of discounting, therefore, is EUR 14,205. The difference still remaining is caused by rounding; i.e. the 1.25 year coupon yield is not exactly 2.9925%.

Figure 8.10 *IRS rate and implied forward yields at the start date of the IRS*



Since the implied forward yields are derived from the spot rates, the strip of forward yields supplemented with the six-month EURIBOR rate is identical to the IRS rate, both giving the same yield. This means that the present value of the two positive cash flows of EUR 4 million from the fixed leg at the moment of closure exactly equals the present value of the four negative cash flows from the floating leg. Thus, the market value of the IRS at the moment of closure is zero. This is shown in figure 8.11.[58]

Figure 8.11 *IRS market value*



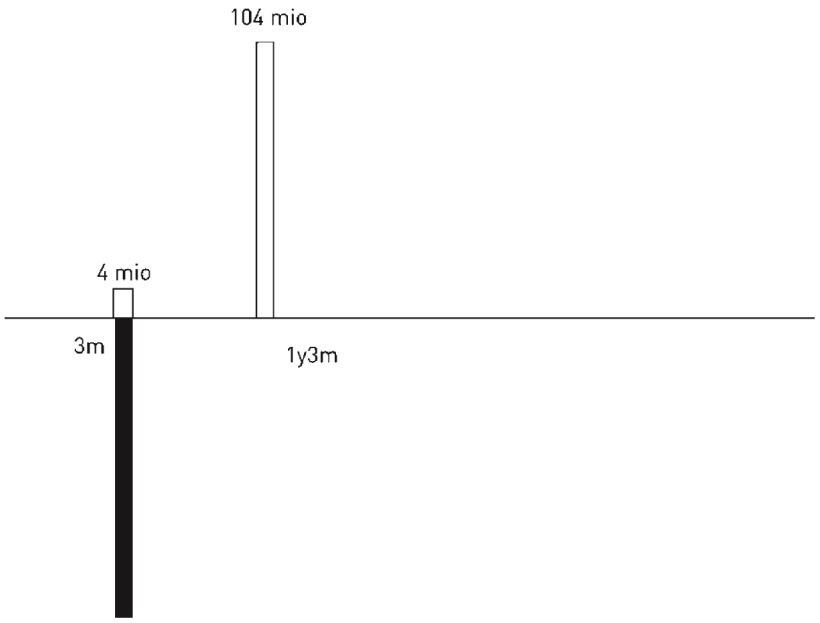
58 The amounts of the floating coupons are calculated as follows:
100 mio x 182/360x 0.0345 = 1,744,167
100 mio x 183/360 x 0.038025 = 1,932,953
100 mio x 182/360 x 0.040951 = 2,070,301
100 mio x 183/360 x0.043004 = 2,186,047

The fact that the market value at the moment of closure is zero applies to any swap and, in general, for each derivative that consists solely of the exchange of future cash flows.

8.7.1 *Modified duration of interest rate swaps*

Just as with other interest instruments, the value of an IRS changes when the interest rate changes. The interest rate sensitivity of the value of an IRS is represented by its modified duration. To calculate the modified duration of an IRS, the IRS can be considered as two opposing loans: a bond and a floating rate note. With this, use can be made of the fact that the value of a floating rate note at a rate fixing moment is always equal to 100 and can therefore be represented by one single cash flow on the next coupon date. This is shown in figure 8.12.

Figure 8.12 *IRS as two opposing loans*



For determining the modified duration of the IRS, the modified duration of the floating leg should be subtracted from the modified duration of the fixed leg (assuming a parallel shift in the yield curve). The modified duration of the floating leg, however, is often left out of the analysis. This is because the price determining parameter for the floating leg is the money market interest rate, which does not have a clear correlation with the capital market interest rate.

EXAMPLE

The following data for a receiver's swap are given:

Remaining term	1,25 years
Fixed rate	4.00%

Suppose that the current one-year and three month IRS rate is also 4.00%. The modified duration of the fixed leg for this IRS can be calculated by using the following equation:

$$\text{Modified duration} = \frac{\frac{\text{COUP}}{r^2} \times \left(1 - \frac{1}{(1+r)^n}\right) + \frac{n \times \left(100 - \frac{\text{COUP}}{r}\right)}{(1+r)^{n+1}}}{\text{price bond}}$$

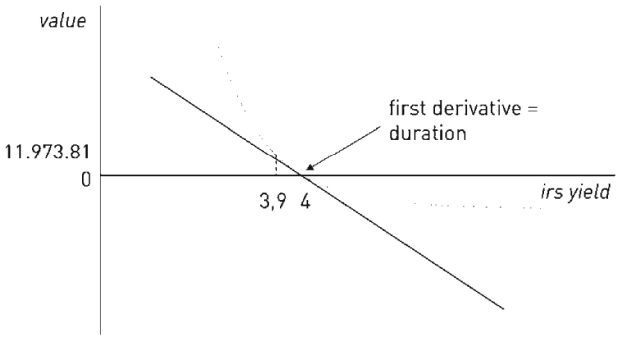
When the data is applied to this equation, it turns out that the modified duration of the fixed leg of the swap is 1.20 (1.196088).

8.7.2 Convexity with interest rate swaps

Figure 8.13 shows the relationship between the value of a receiver's swap with a coupon of 4% and a remaining term of 1.25 years and the long term interest rate. The nominal value of the IRS is EUR 10,000,000. The figure shows that the value of the IRS with the current IRS rate of 4% is zero. For IRS rates below 4% the swap has a positive value while for rates above 4% the swap has a negative value.

Just as with bonds, the relationship between the interest rate and the value of an IRS is not linear. Receiver's swaps can generally be compared with bonds and the graph for a receiver's swap also shows a positive convexity This means that, in the case of falling interest rates, the value of a receiver's swap increases faster than would be expected given the modified duration. In the case of rising in interest rates, the value of the swap decreases to a lesser degree than would be expected given the modified duration.

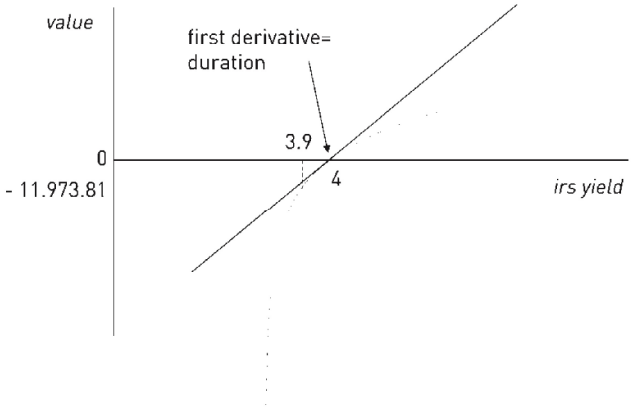
Figure 8.13 *Relationship between the interest rate and the value of a receiver's swap*



The duration of this receiver's swap is 1.196088. If the relationship between the price of the IRS and the yield were linear, a decrease in the interest rate from 4% to 3.9% would lead to an increase in the value of the swap of 1.196088% – thus EUR 11,960.88 (10,000,000 x 0.001 x 1.196088). However, in reality, the value of the swap increases by EUR 11,973.81, which is higher.

For payer's swaps, the convexity is negative. In figure 8.14 the relationship between the interest rate and a payer's swap with a fixed rate of 4% and a remaining term of 1.25 years is shown.

Figure 8.14 *Relationship between the interest rate and the value of a payer's swap*



Because of the negative convexity, the decrease in value of the swap as a result of falling interest rates is underestimated by the modified duration and the increase in value as a result of rising interest rates is overestimated. For clients who have concluded a payer's IRS and who use the modified duration to indicate the changes in

the market value of this IRS, this is unpleasant news. The change in the value of the swap, both for a fall or a rise in interest rates, is worse than they expect.

8.7.3 The basis point value of interest rate swaps

The modified duration can also be used to calculate the basis point value of an IRS. At the time of concluding an IRS, the BPV is determined as follows:

bpv interest rate swap = principal of the irs × modified duration × 0.0001

During the term of an IRS, the BPV is calculated as follows:

bpv interest rate swap = (principal + market value irs) × modified duration × 0.0001

Traders in interest derivatives use the BPV to gain an insight into the risk of their position. Corporate dealers can use the BPV to calculate the margin that they achieve on an IRS transaction that they have concluded with a client.

EXAMPLE

A corporate dealer concludes a swap with his client for a nominal amount of EUR 10 million, a term of 5 years and a fixed interest rate of 4.5%. The modified duration of the IRS is 4.39.

The basis point value of this IRS is

EUR 10,000,000 x 4.39 x 0.0001 = EUR 4,390.

The corporate dealer has taken a margin of three basis points above the price he has obtained from the interest rate derivatives dealer. Expressed in euro, this is a margin of 3 x EUR 4,390.00 = EUR 13,170.00.